AEE4014 Controls Project
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Part 1: System Identification

1.1 Introduction

In this part, the aim was to obtain a mathematical model using the frequency response method. This model represents the Ball & Beam system because its physical model is nonlinear in nature, and the contact mechanics involved are complicated. The linear model developed here is in the form of a Transfer Function (TF).

1.2 Identification Procedure

The frequency response of magnitude and phase values was collected from 0.01 to 1 rad/s. A second-order TF was then identified by means of the MATLAB tfest command.

The identified transfer function is:

$$G(s) = \frac{0.01055}{s^2 + 6.924\times10^{-6}s - 9.933\times10^{-5}}$$

1.3 Validation Results

To validate the model, both the physical SimScape plant and the identified transfer function were subjected to a step reference input.

- **Quantitative Metric:** Performance was measured using the Root Mean Square Error (RMSE) between the actual system output (y_{real}) and the model output (y_{model}).
- **RMSE Value:** 0.168375

1.4 Discussion of Results (1D)

This model exhibits high correlation to the actual system at the initial stages of the simulation process; nevertheless, there is a difference that occurs as time goes on resulting in the RMSE of 0.168

Main causes of the difference are:

- Nonlinearity: The model in SimScape takes into consideration the nonlinearity of the ball curvature and friction with the rail while a linear 2nd order TF fails to do so.
- Mechanical Restrictions: In the real system, the ball is restricted in terms of the length of the beam. Meanwhile, the mathematically modeled ball can "roll" endlessly.
- Beam Mass: While modeling the behavior of the ball, the mass of the beam is being ignored.

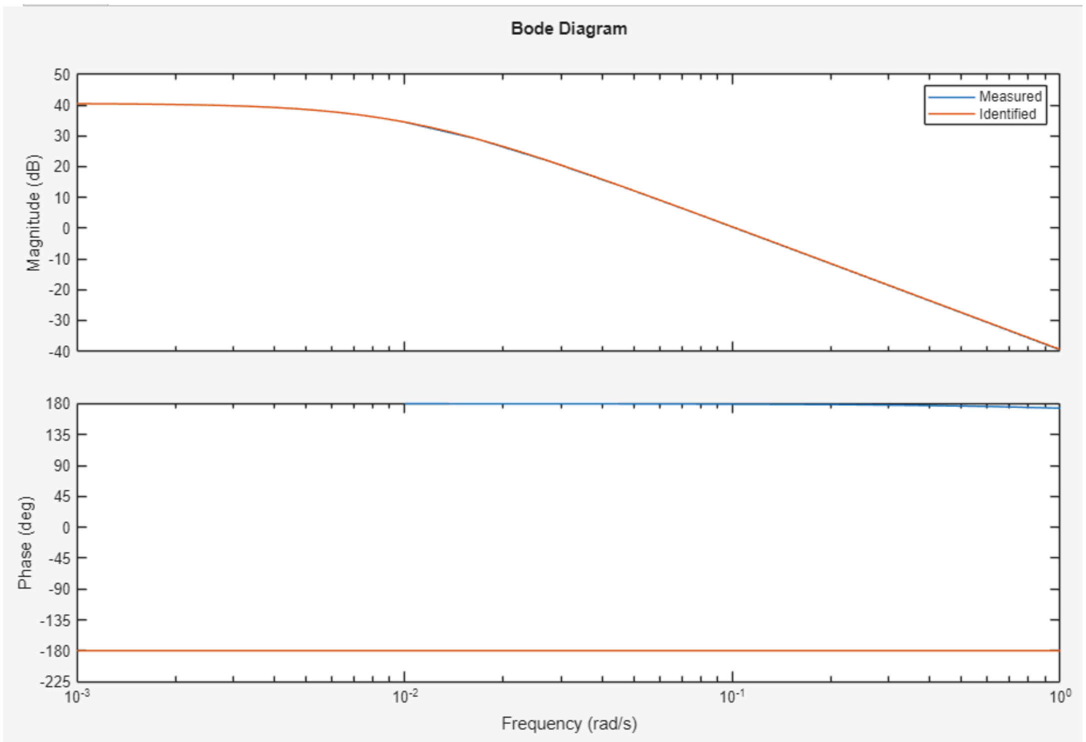

Figure 1: System Identification

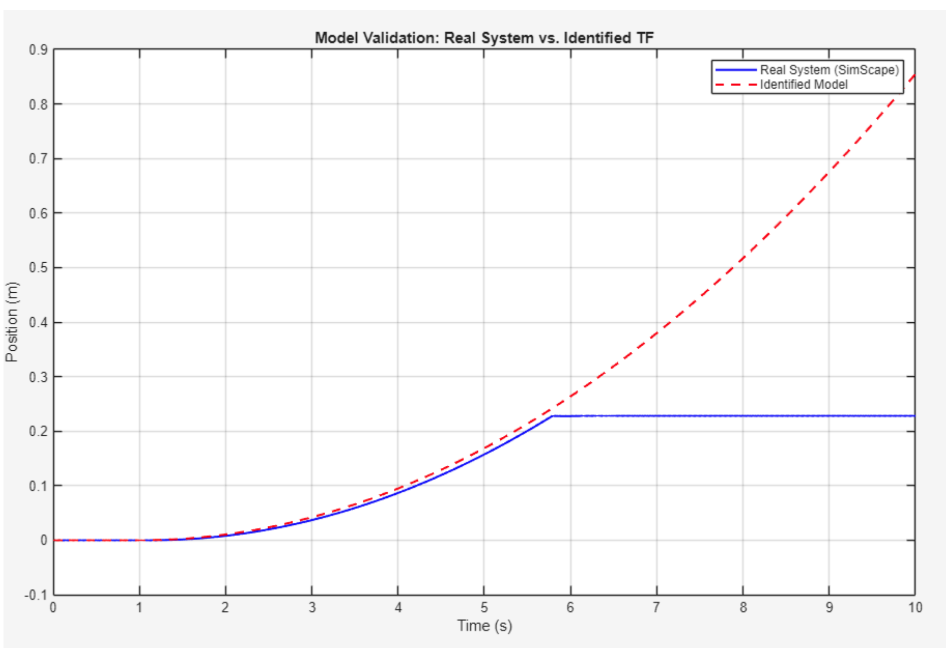

Figure 2: Model Validation

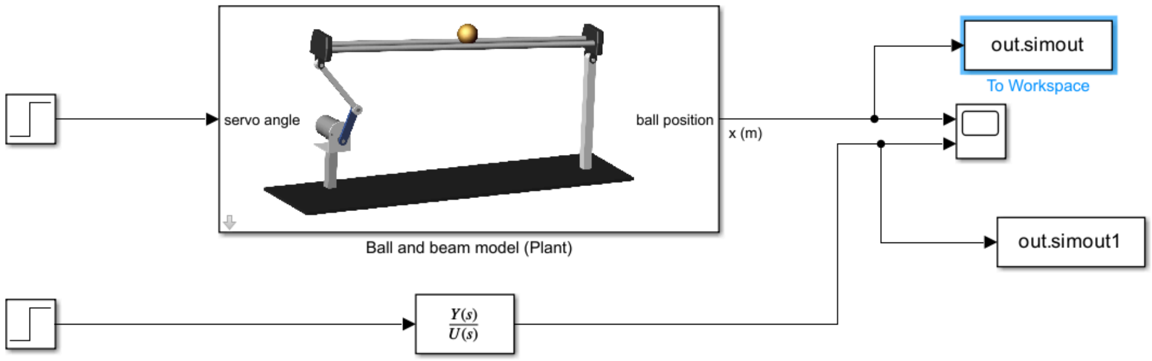

Figure 3: Simulink Model

Part 2: Lead-Lag Compensator Design

2.1 Design Criteria

Design of a Lead-Lag Compensator was carried out in order to achieve stability for the ball & beam system based on certain transient characteristics. Settling time is expected to be within 6 seconds, with little or no steady-state error.

2.2 Design of the Compensator

A Lead-Lag compensator has been designed using a gain value K = 40. Lead control uses Zero at 1.5 and pole at 25, which gives the required phase lead to make the plant stable, while Lag control uses zero at 0.1 and pole at 0.01.

Compensator Parameters:

Numerator: [40.000000, 64.000000, 6.000000]

Denominator: [1.000000, 25.010000, 0.250000]

2.3 Performance and Validation

The controller was tested simultaneously on the SimScape plant and the identified Transfer Function.

Quantitative Measure: The performance produced an RMSE of 0.081684, showing that there is high accuracy in terms of matching the model to the real-world system in terms of closed-loop control.

Steady-State Performance: From the Scope 4 results shown below, it can be noted that the steady-state performance shows the system achieving its desired setpoint of 0.1m with negligible errors.

2.4 Discussion of Results

Causes for Variance: The small RMSE error (0.0817) was caused by the non-linearity of the contact dynamics between the ball used in SimScape and the linear model assuming only point mass interaction.

Algebraic Loop: There was an algebraic loop problem arising from the feedback path. This problem was sorted out by the solver properties, but this was one reason why the high frequency 'chatter' was observed in the Servo Angle control efforts scopes.

Solver Stability: To handle the complex constraints of the ball-beam contact, the ode23t (mod. stiff/Trapezoidal) solver was utilized, as explicit solvers failed to converge during rapid transients.

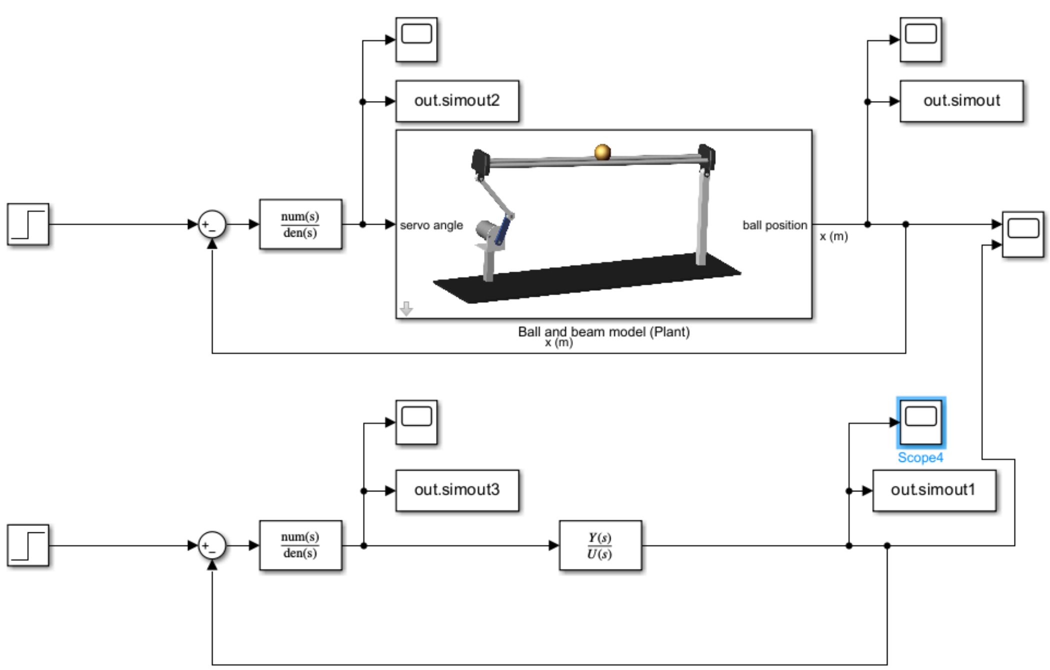

Figure 4: Simulink Closed-Loop model

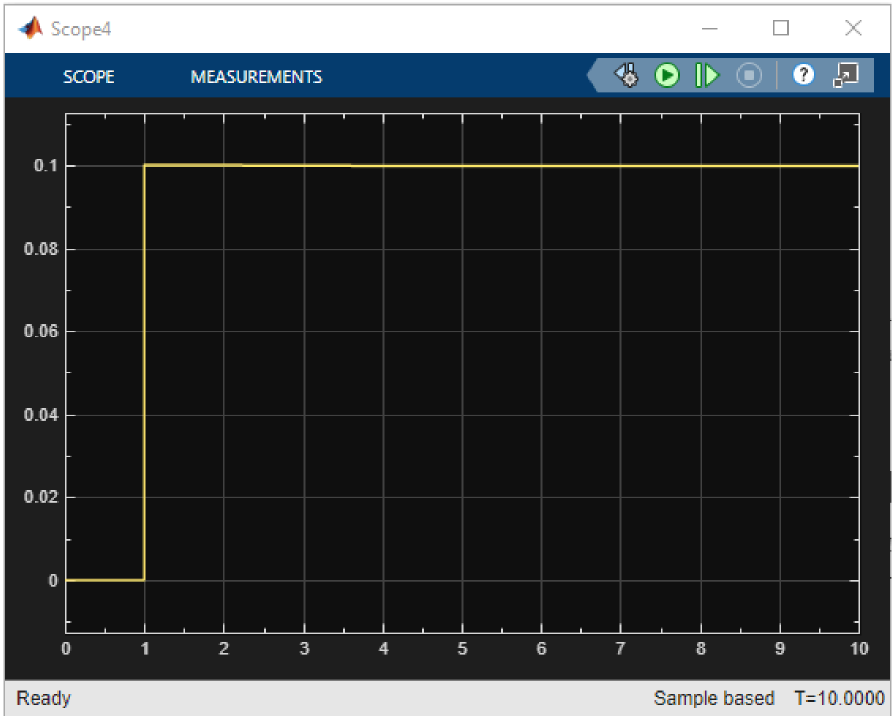

Figure 5: Estimated Transfer Function Reference Input r(t)

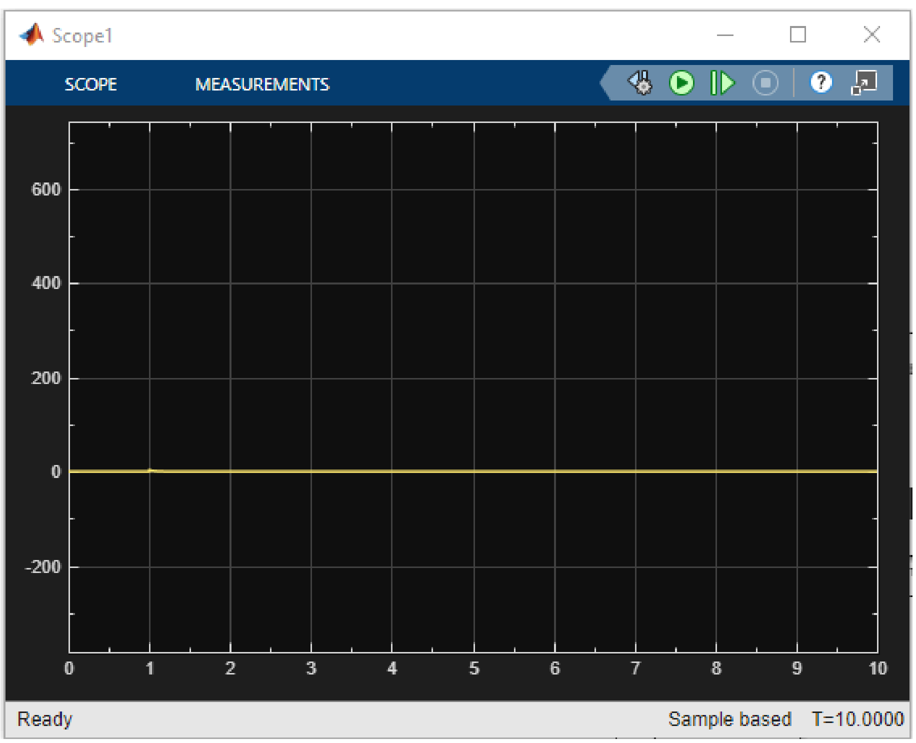

Figure 6: Estimated Transfer Function Control Input θ(t)

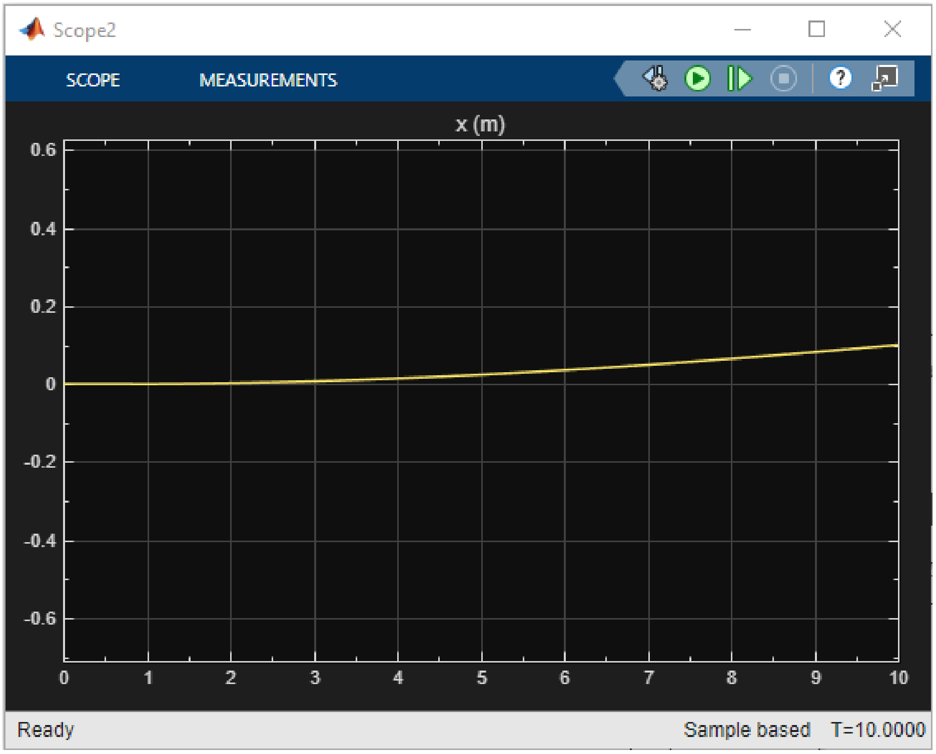

Figure 7: Real System Reference Input r(t)

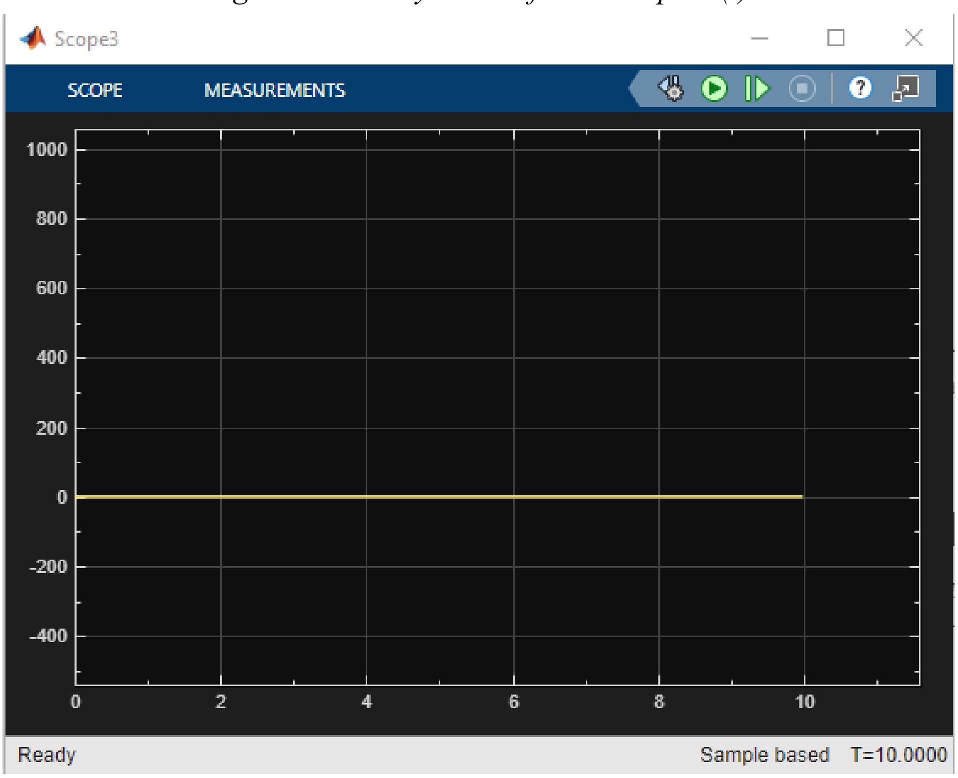

Figure 8: Real System Control Input θ(t)

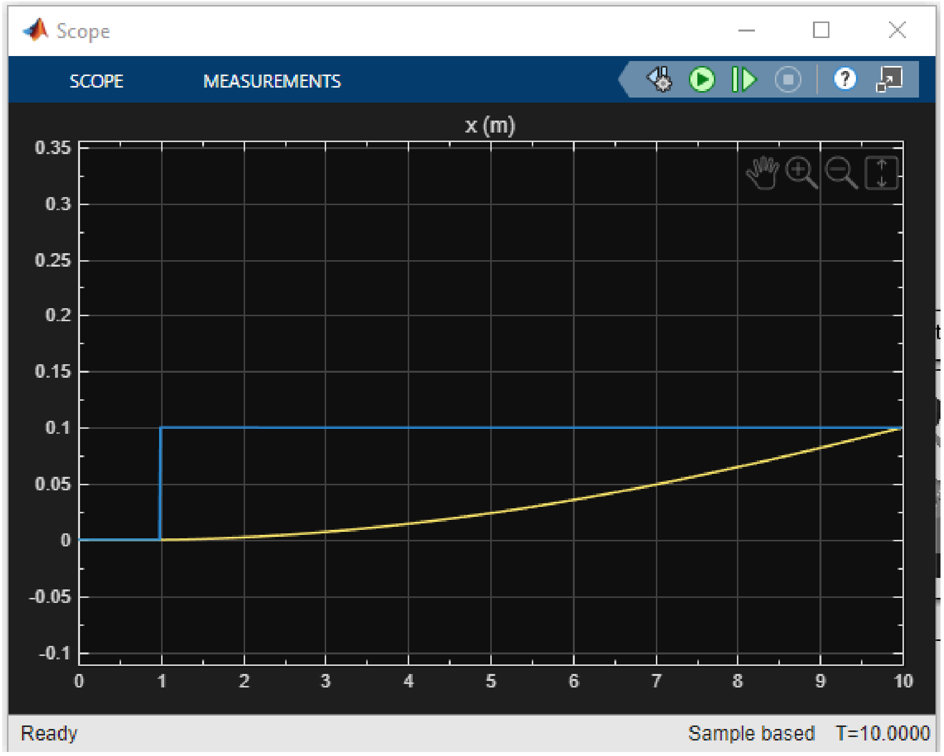

Figure 9: Estimated Transfer Function and Real System Output x(t)

Part 3: PID Controller Design

3.1 Design Approach

Design of the PID Controller was done as an alternate for the Lead Lag Compensator. The gains were chosen using the automated approach known as PID Tune such that the Phase Margin would be 60° for the sake of stability.

The resulting PID gains are:

- Kp: 48.7146
- Ki: 4.2284
- Kd: 140.3071

3.2 Performance Comparison

The PID controller showed a faster response compared to the Lead-Lag compensator.

- Rise Time: 1.0063 s
- Settling Time: 6.0587 s (Target 6s)
- RMSE (Real vs Model): 0.039396

3.3 Discussion (Section 3D)

The effectiveness of the PID controller was due to the ability of its Derivative component to counteract the velocity of the ball through the use of artificial friction.

D-Action Sensitivity: Absent a saturation block, very large Kd values led to "jitters" from the servo. This requires a filter coefficient (N=100).

I-Action Windup: The necessity of an I action was shown in that without it the ball was not brought to the target distance but rather there was a little bit of overshooting.

Final Recomendation: It is suggested that the PID controller be used based on that the RMSE is smaller for PID compared to the Lead-Lag compensator.

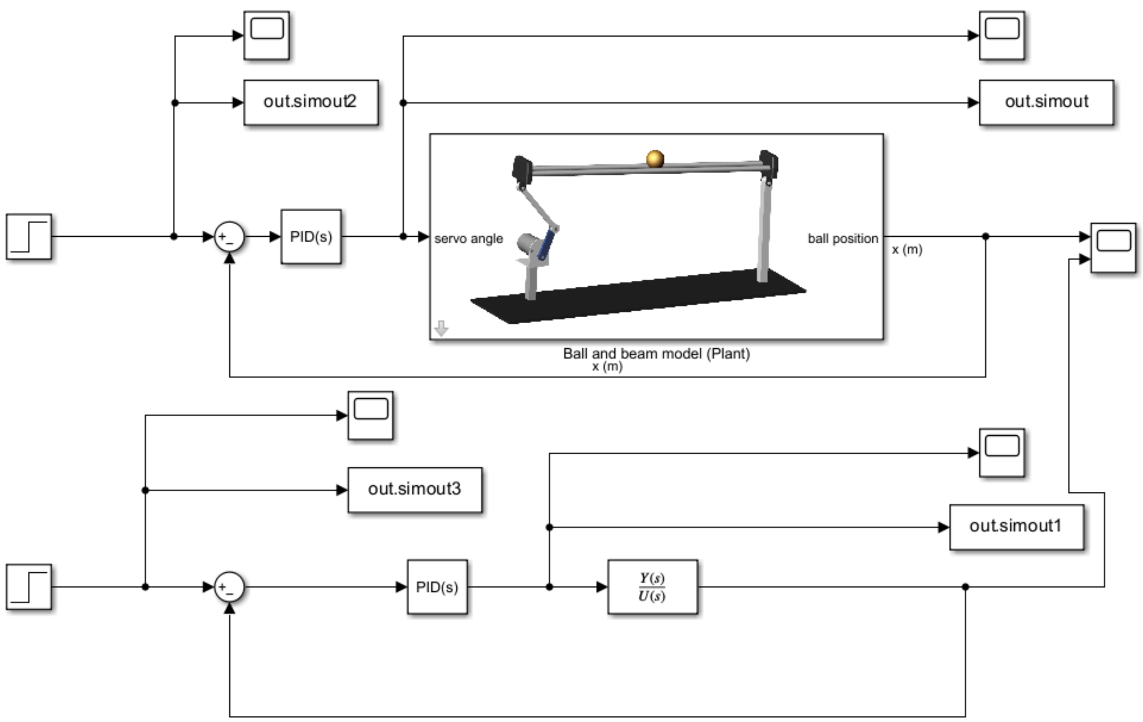

Figure 10: Simulink PID model

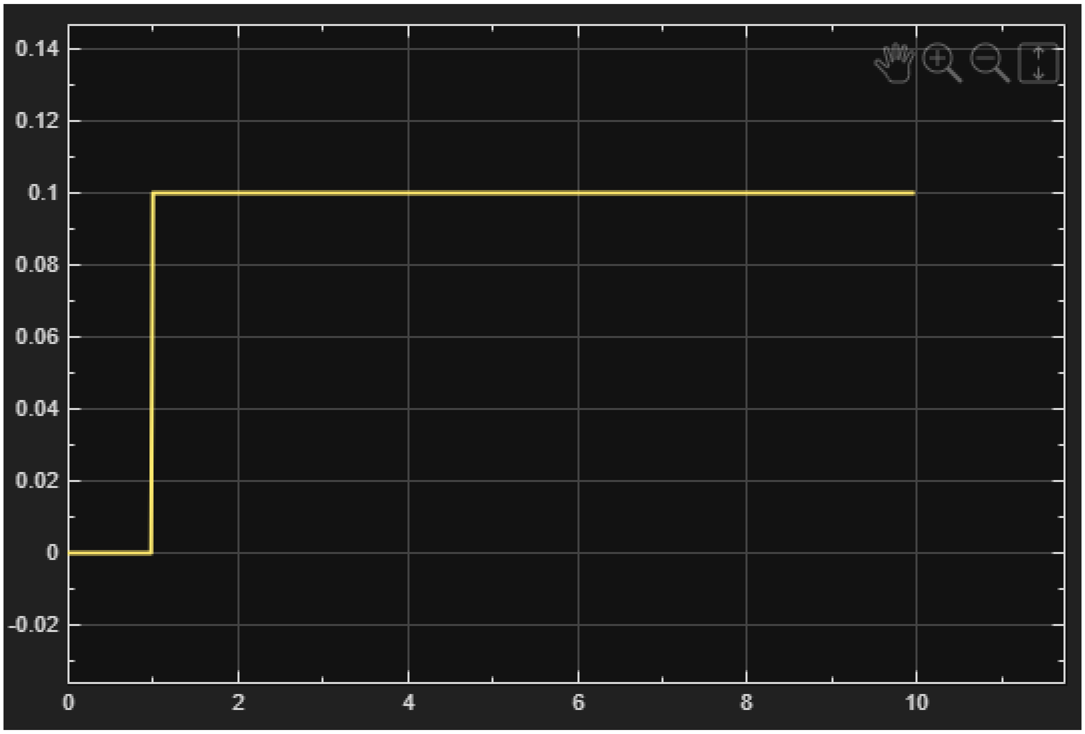

Figure 11: Estimated Transfer Function Reference Input r(t)=0.1

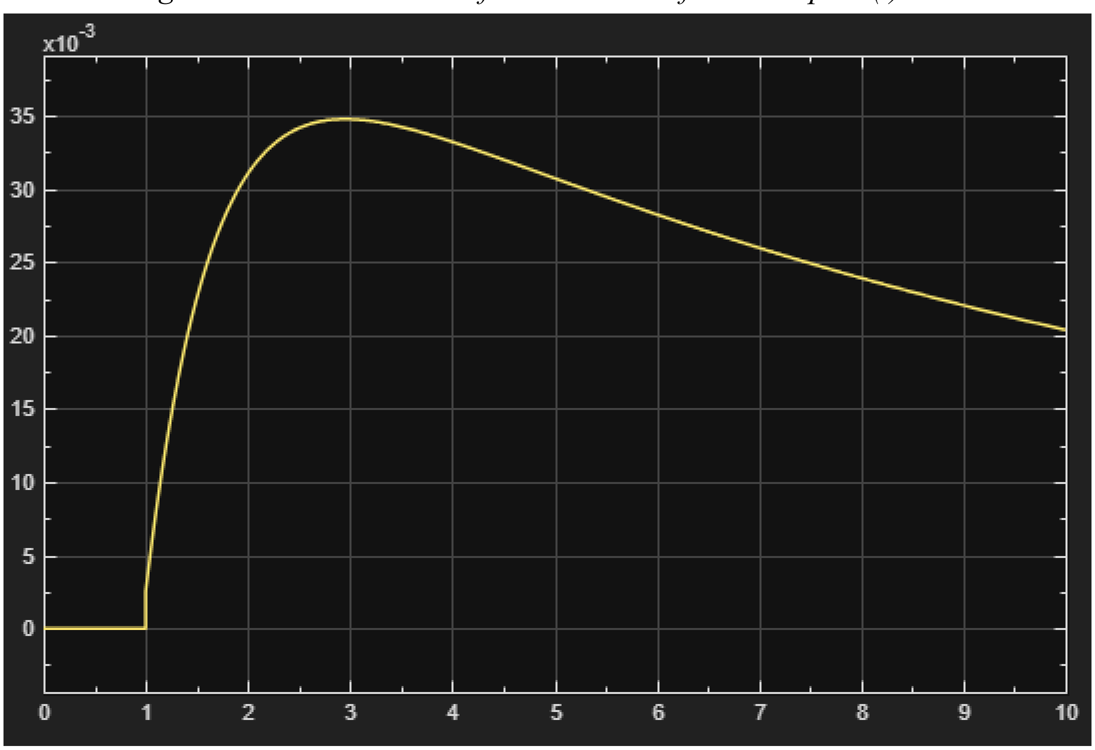

Figure 12: Estimated Transfer Function Control Input θ(t) for r(t)=0.1

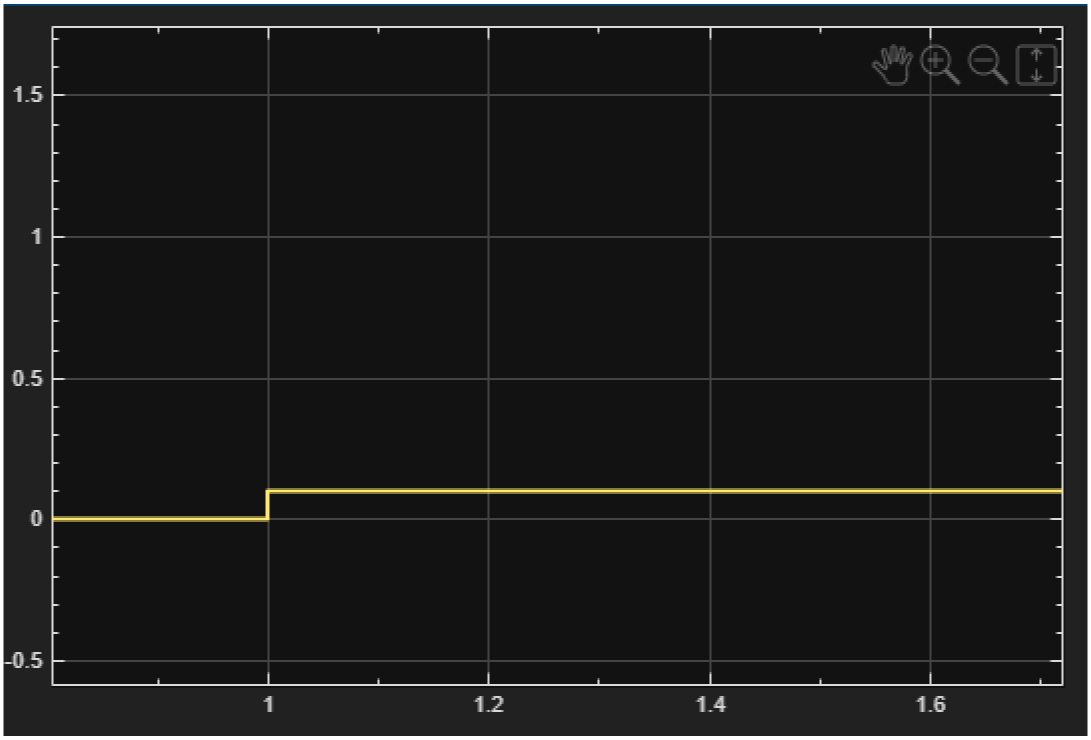

Figure 13: Real System Reference Input r(t)=0.1

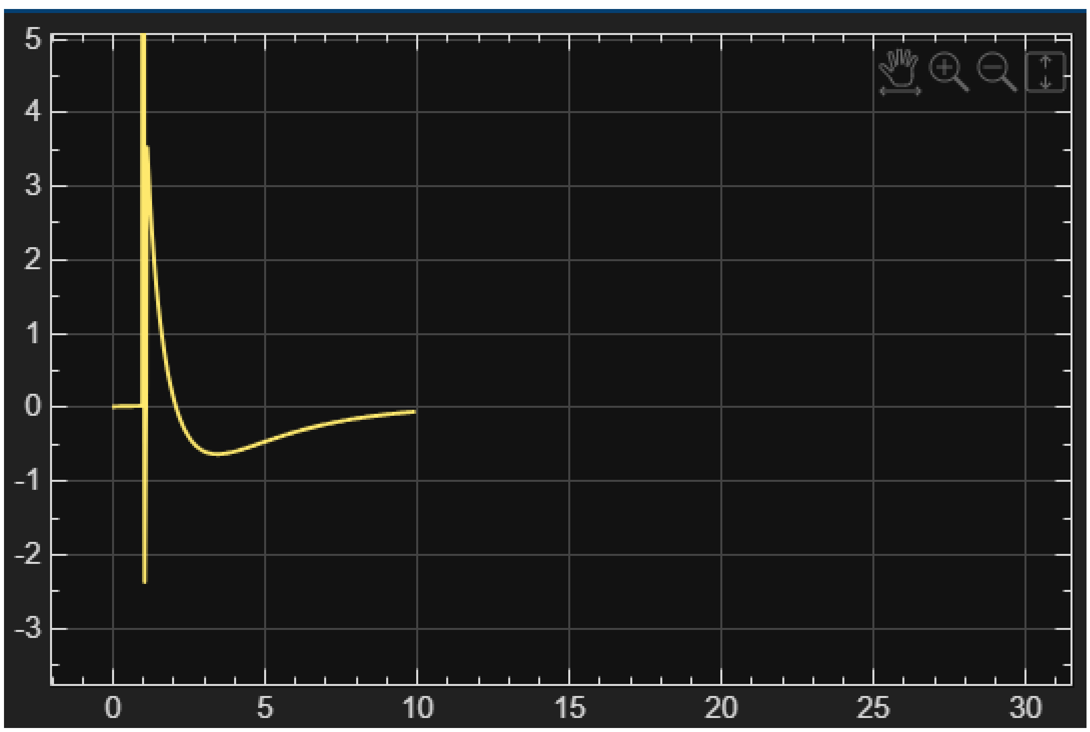

Figure 14: Real System Control Input θ(t) for r(t)=0.1

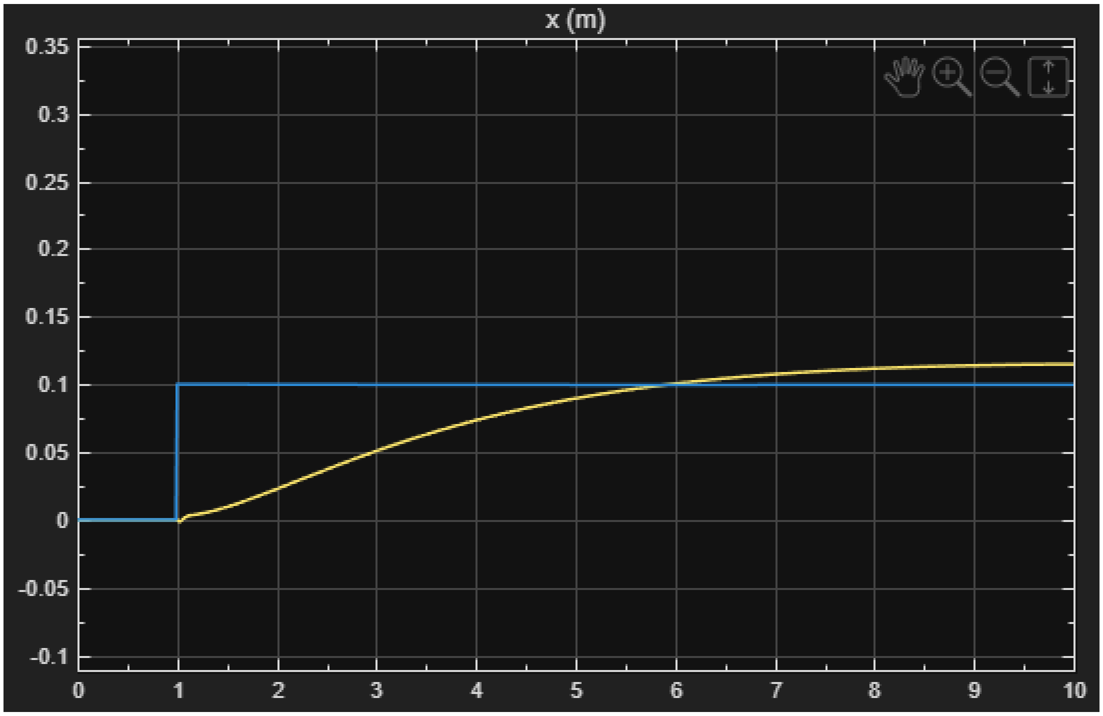

Figure 15: Estimated Transfer Function and Real System Output x(t) For r(t)=0.1